SUBJECT:
OPTICAL COMMUNICATION
SUB. CODE:
BEC057
BRANCH: ECE
SEM: 5TH

EC-Vth

BEC-057 OPTICAL COMMUNICATION (OC)

① Introduction to optical Communication

General Communication System

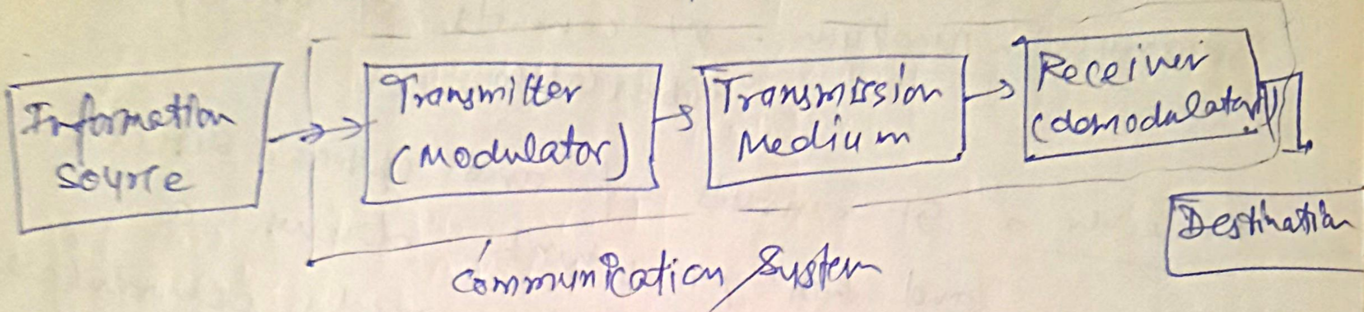
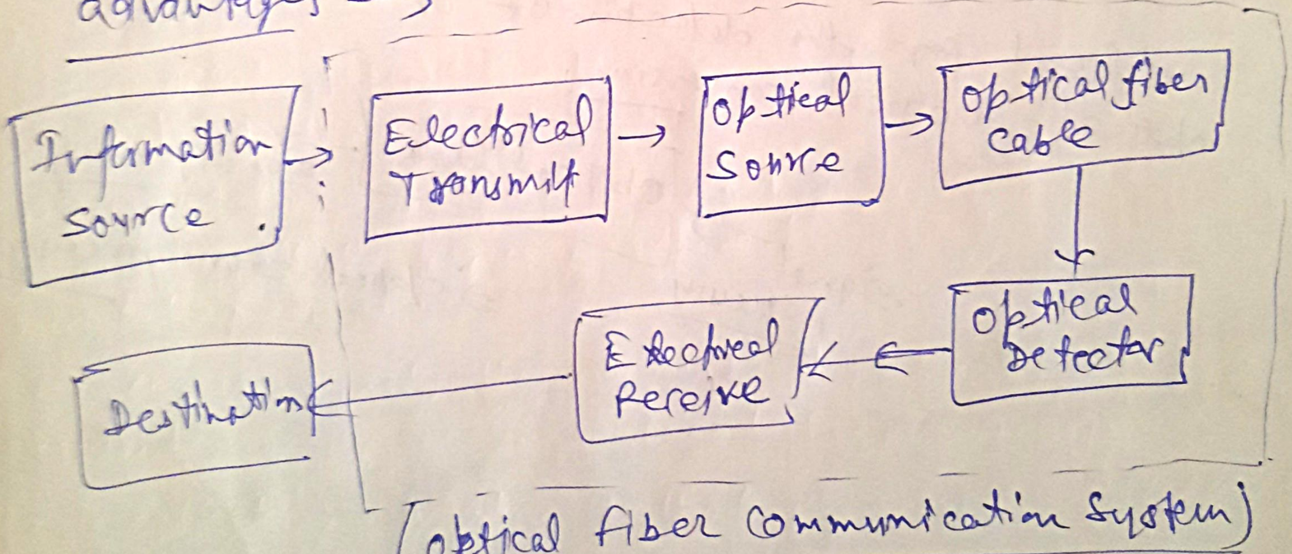


fig: General communication system

② Optical Communication system with its advantages →



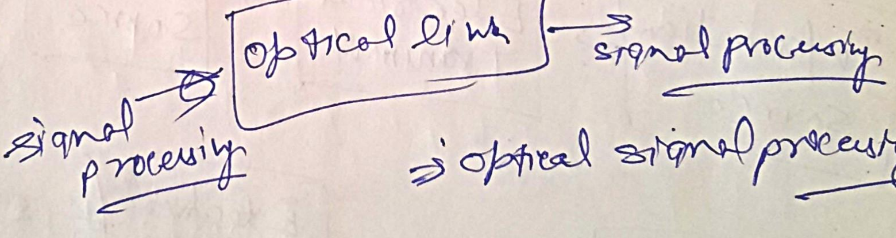
Information source provides an electrical signal to a transmitter comprising an electrical stage which drives an optical source to give modulation of the light wave carrier.

optical Source → It provides an electrical-optical conversion by a semiconductor laser or LED (Light Emitting Diode).

Transmission Medium - It consists of an optical fiber cable.

Receiver → It consists of and optical detector ~~and hon pr~~ which drives a further electrical stage and provides demodulation of optical carrier by photodiode (p-n, p-i-n. or avalanche)

sometimes photo transistor and photoconductors are utilized for the detection of optical signal and optical-electrical conversion.



⇒ optical signal processing

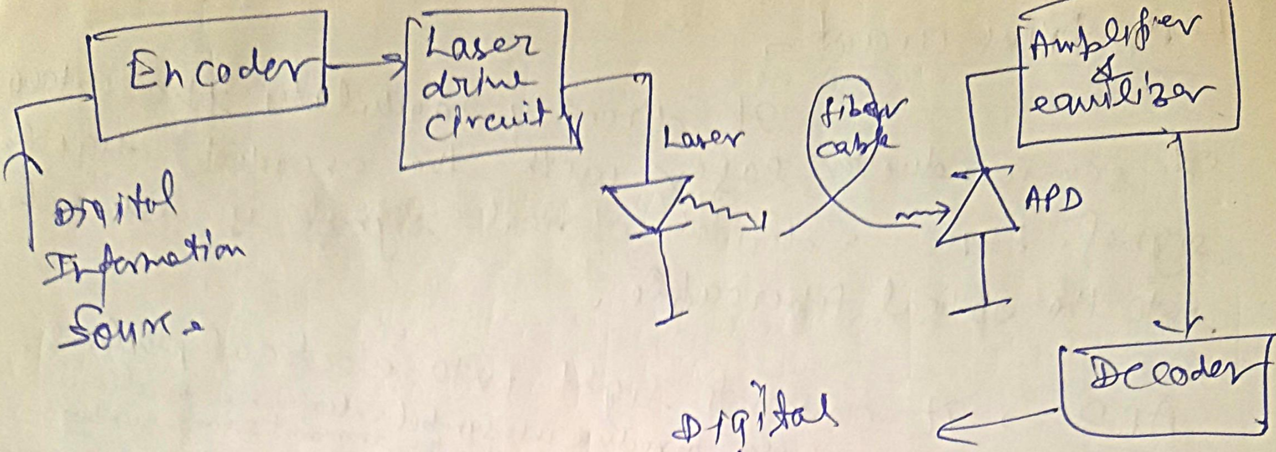


Fig: A digital optical fiber link using a semiconductor laser source and an avalanche photodiode (APD) detector.

An analog modulation - the variation of light emitted from the optical source is in continuous manner.

In Digital Modulation - the discrete change in the light intensity are obtained. (ON. OFF pulse)

⇒ Analog optical fiber communication link are generally limited to shorter distance and lower bandwidths than digital links.

because of Analog is less efficient, & ~~higher~~ ~~S/NR~~ Noise, linearity problem

⇒ Encoder → The input digital signal from the information source is suitably encoded for optical transmission

Laser Drive circuit :

It directly modulates the intensity of semiconductor laser with the encoded digital signal. Hence a digital optical signal is launched into the optical fiber cable.

APD → It converts light into electrical pulses.

→ filter to reduce intersymbol interference to allow recovery of the transmit ~~signal~~ symbols.

Amplifier & Equalizer — It provides signal processing to find gain and make bandwidth reduction

$\boxed{\text{Decoder}}$ → signal obtained is decoded to give the original digital information.

Advantages of optical fiber communication →

(a) Enormous potential Bandwidth :

co-axial cable BW → ≈ 500 MHz

optical carrier freq ⇒ 10^{13} to 10^{16} Hz

(≈ nearly infrared 10^{14} Hz — 15 GHz)

⇒ Many more optical signals ~~are~~ may be transmitted.

(b) Small size and weight :

— diameter of optical fiber < diameter of human hair

after protective coating, they are lighter than copper cable.

⇒ use → aircraft, satellites, ships.

(c) Electrical Isolation
↓
It is fabricated with glass or plastic polymer. So no interface problem and sparking problem.

(d) Immunity to interface and cross talk
• There is ~~no~~ optical interference between fibers.
~~and~~ ~~negligible~~ and negligible cross talk.

(e) signal security - There is signal security in optical fiber so used in military, banking & computer n/w.

(f) Low transmission loss -
↳ ≃ 0.2 dB/km.

⇒ It gives reduction in system cost and complexity.

(g) Ruggedness and flexibility
↓
It may bend to quite small raddi and twisted without damage
→ compact and hard

⇒ ~~easy~~ superior in terms of storage, transportation, handling and installation than copper cables

(h) System Reliability and Ease of Maintenance

(Proper working)

Reliability of the optical components is no longer a problem with predicted lifetimes of 20 to 30 years.

⇒ reduce maintenance time & cost.

(i) Potential Low Cost - glass ⇒ made of sand

cost of optical fiber is not very ~~costly~~ high

for long-distance communication overall system cost of optical fiber communication is less than equivalent electrical line systems.

Advantages of OFC

(1) BW is High

(2) size & weight : small

(3) electrical isolation

(4) Immunity to interference & cross talk

(5) signal security

(6) Low transmission Loss

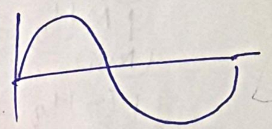
(7) Ruggedness & flexibility

(8) System reliability & ease of maintenance

(9) Low cost.

Band	Freq Range
HF	→ 3 - 30 MHz
VHF	→ 30 - 300 MHz
UHF	→ 300 - 3000 MHz
L	→ 1 - 2 GHz
S	→ 2 - 4 GHz
C	→ 4 - 8 GHz
X	→ 8 - 12 GHz
Ku	→ 12 - 18 GHz
K	→ 18 - 27 GHz
Ka	→ 27 - 40 GHz
V	→ 40 - 75 GHz
W	→ 75 - 110 GHz
mm	→ 110 - 330 GHz

v = n λ



λ : higher

n : ↓

n : f ⇒ λ : ↓

OFC → (1.7 μm — 0.8 μm)

Visible spectrum:

Red

≥ 0.7 μm

Violet

0.4 μm

LED - BW → 50 - 140 MHz

white light → ≤ 85 MHz

$\nu \rightarrow$	30 EHz	30 PHz	790 THz	430 THz	300 GHz	300 MHz
	Gamma	X ray	Ultraviolet		IR	Micro wave	Radio
λ	10 pm	10 nm	380 nm	780 nm	1 mm	1 m

V	I	B	G	Y	O	R

380 nm, 400, 450, 500, 550, 600, 650, 700, 780 nm

$1\ KHz = 10^3\ Hz$ (Kilo)

$1\ MHz = 10^6\ Hz$ (Mega)

$1\ GHz = 10^9\ Hz$ (Giga)

$1\ THz = 10^{12}\ Hz$ (Tera)

$1\ PHz = 10^{15}\ Hz$ (Peta)

$1\ EHz = 10^{18}\ Hz$ (Exahertz)

RF spectrum

VLF → 3 KHz – 30 KHz

LF → 30 KHz – 300 KHz

MF → 300 KHz – 3 MHz

HF → 3 MHz – 30 MHz

VHF → 30 MHz – 300 MHz

Ultra UHF → 300 MHz – 3 GHz

Super SHF → 3 GHz – 30 GHz

extremely EHF → 30 GHz – 300 GHz

optical spectral Band with operating windows.

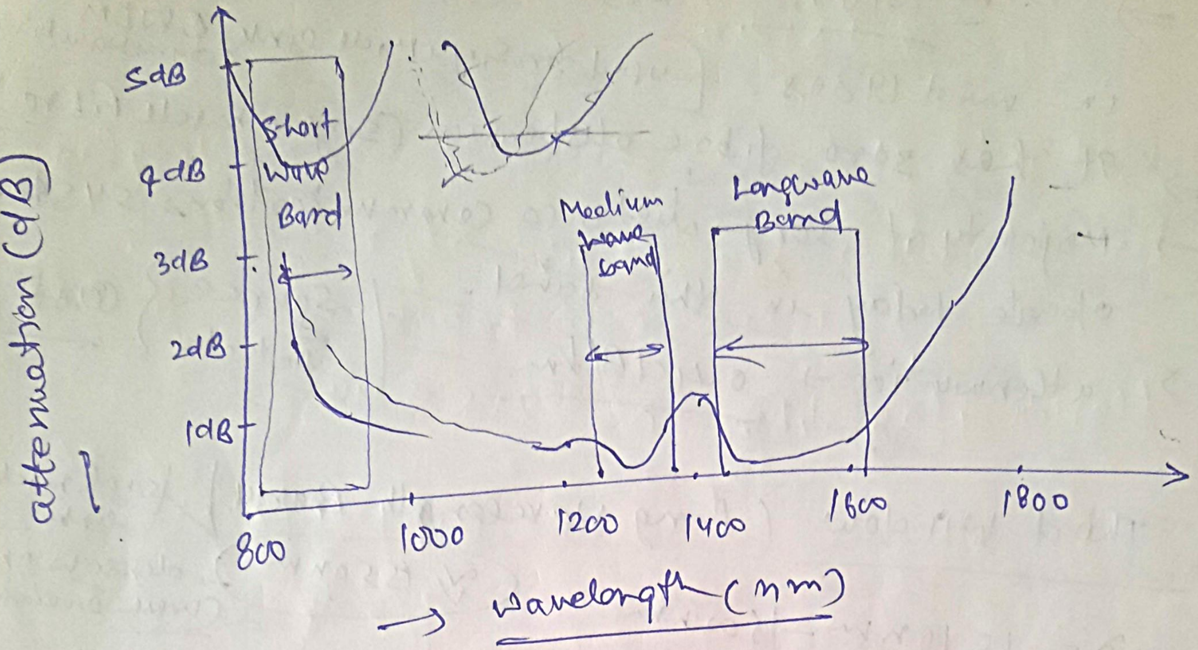


Fig: Transmission windows -

The upper curve of shows the absorption characteristic of ~~of~~ fibre in 1970s. The lower one is for modern optical fibre.

⇒ There are three windows or bands in the transmission spectrum of optical fibre.

⇒ The wavelength band used by a system is an extremely important defining characteristic of that optical system.

(#) First window - - short wavelength Band

≈ 850nm - used for multimode link.

λ = 800 - 900 nm

This was the first Band used for optical fibre communication in the 1970s. and early 1980s. It was attractive because of low cost optical sources and detectors in this band.

Second Window (Medium wavelength Band)

⇒ ≃ 1310 nm which came into existance in mid 1980s. (used for single mode link, CWDM, course wave Div Multi)

→ It has zero fibre dispersion (single mode fibre)

→ Majority of long distance communications systems operate today in this band.

→ attenuation → 0.4 dB/Km

| Source & Detector } costly

Third Window (Long wavelength Band)

≃ 1550 nm → (single mode link) → dense WDM (wave Division Multi)

⇒ λ = 1510 nm – 1600 nm

→ attenuation ≃ 0.26 dB/Km

→ optical Amplifier is used

→ after 1990s is used.

| Source & Detector) costly.

attenuation → amount of light loss b/w input & output. It is the sum of all losses in fibre.

unit → dB/Km (decibel/Km)

Optical fiber Waveguide

① Ray ~~Transmission~~ Theory of Transmission with TIR. (Total Internal Reflection)

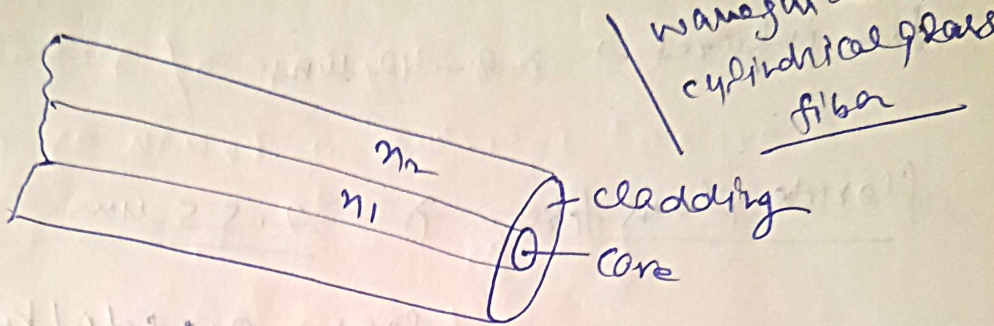


Fig: optical fiber waveguide showing the core of refractive index n_1, surrounded by the cladding slightly lower refractive index n_2

$n_1 > n_2$

⇒ Light energy travels in both core & cladding decay is negligible at cladding-air interface.

⇒ cladding support the waveguide structure

1966 ⇒ attenuation Loss - 1000 dB/km

→ 4.2 dB/km by ~~refining~~ conventional glass refining tech.

→ 1 dB/km

In first Generation. gallium aluminium Arsenide alloys

→ $\lambda = 0.8 - 0.9\,\mu m$

$\lambda \rightarrow 1.1 - 1.6 \mu m \Rightarrow$ loss $= 0.2 \, dB/Km$

$\simeq 1.55 \mu m$

mid infrared $\rightarrow \lambda = 2-5 \mu m$ } silicate glass.

far infrared $\rightarrow \lambda = 8 - 12 \mu m$

florride glass fiber. $\lambda = 2.55 \mu m$

. loss $= 0.01 \, dB/Km$

step & graded index fiber

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refractive index of a Medium $\rightarrow$ ratio of the velocity of light in a vaccum to the velocity of light in the medium.

$$\eta = \frac{V_d}{V_m}$$

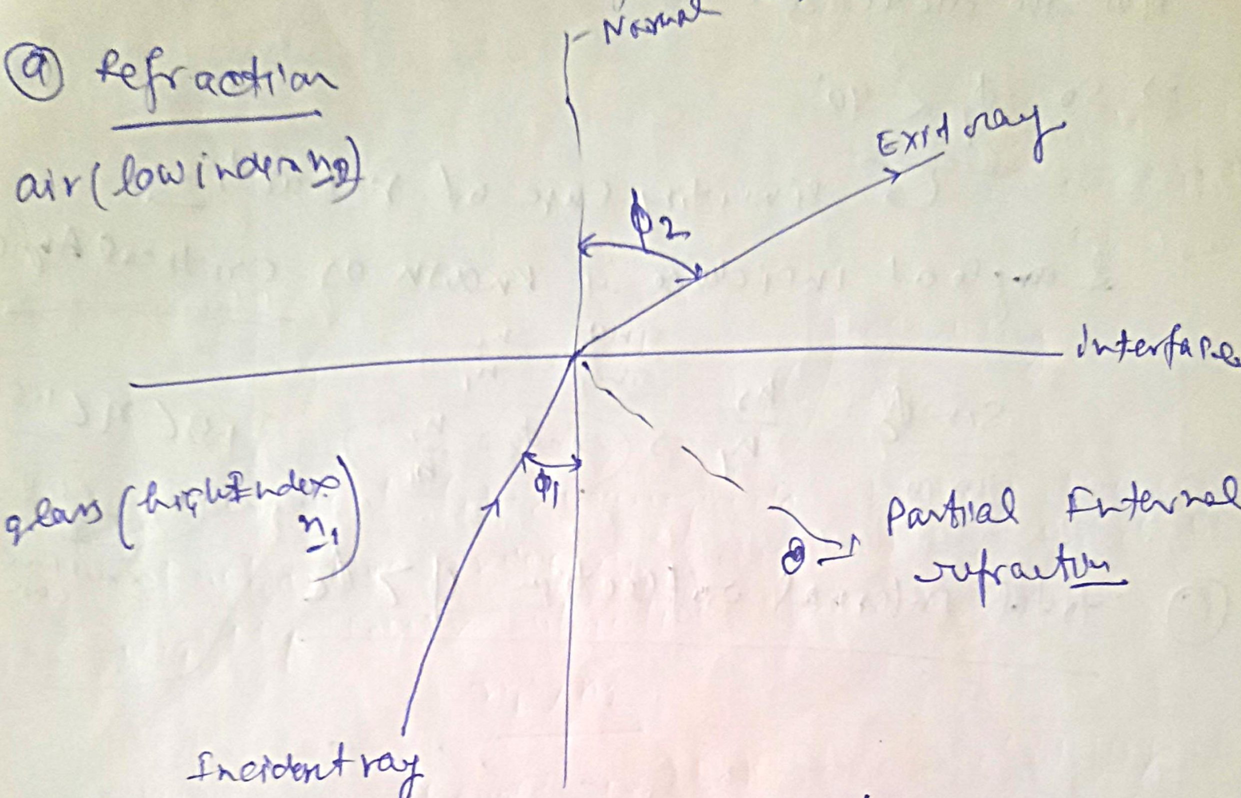
$(V)_{dense} < (V)_{rare}$ hence the η give measure of this effect.

dielectrics - materials possessing high electrical resistivity.

$\rightarrow$ good insulator

Light ray incident on high to low refractive index interface (glass – air)

(a) Refraction



$\phi_1 \rightarrow$ angle of incidence

$\phi_2 \rightarrow$ angle of refraction

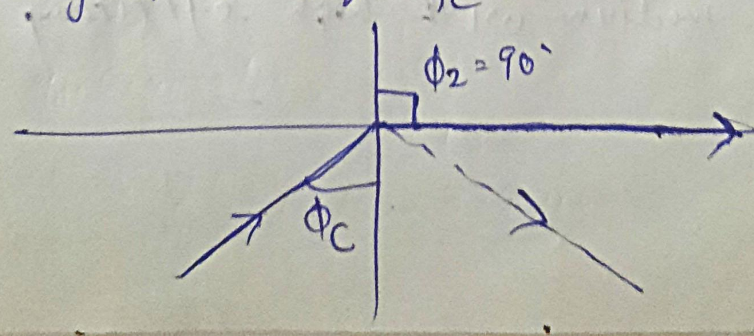
$\phi_2 > \phi_1$

Snell's Law of refraction

$$n_1 \sin\phi_1 = n_2 \sin\phi_2$$

$$\frac{\sin\phi_1}{\sin\phi_2} = \frac{n_2}{n_1}$$

(b) The limiting case of refraction showing the critical ray at an angle ϕ_C.



when ϕ_2 (angle of refraction) $= 90°$

$\Rightarrow$ the refracted rays emerge parallel to the interface b/w the dielectrics.

$\Rightarrow$ so $\phi_1 < 90°$

↳ limiting case of refraction.

& angle of incidence is known as critical Angle

$$\frac{\sin\phi_1}{\sin\phi_2} = \frac{n_2}{n_1}$$

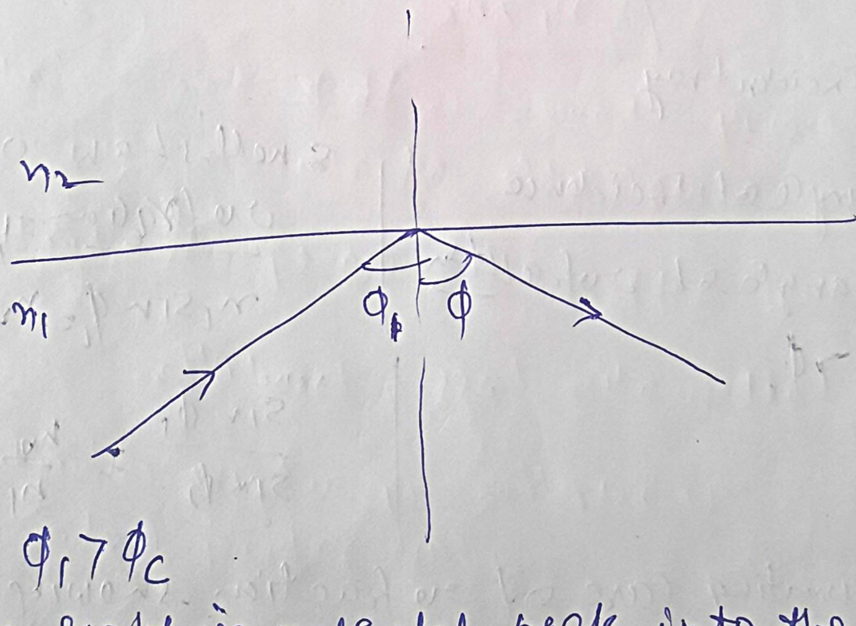
$$\frac{\sin\phi_c}{\sin 90} = \frac{n_2}{n_1} \Rightarrow \sin\phi_c = \frac{n_2}{n_1}$$

$1.33 < \eta < 1.50$

$\eta_{x\text{-ray}} < 1$

exceptional case

(c) total internal reflection $\phi > \phi_c$



when $\phi_1 > \phi_c$

$\rightarrow$ the light is reflected back into the originating dielectric medium with high efficiency (99.9%)

$\rightarrow$ TIR

$\Rightarrow$

total internal reflection occurs at the interface b/w two dielectrics of different refractive indices when light is incident on the dielectric of lower index from the dielectric of higher index and angle of incidence of the ray exceeds the critical value.

shallow angle → $90 - \phi_c$

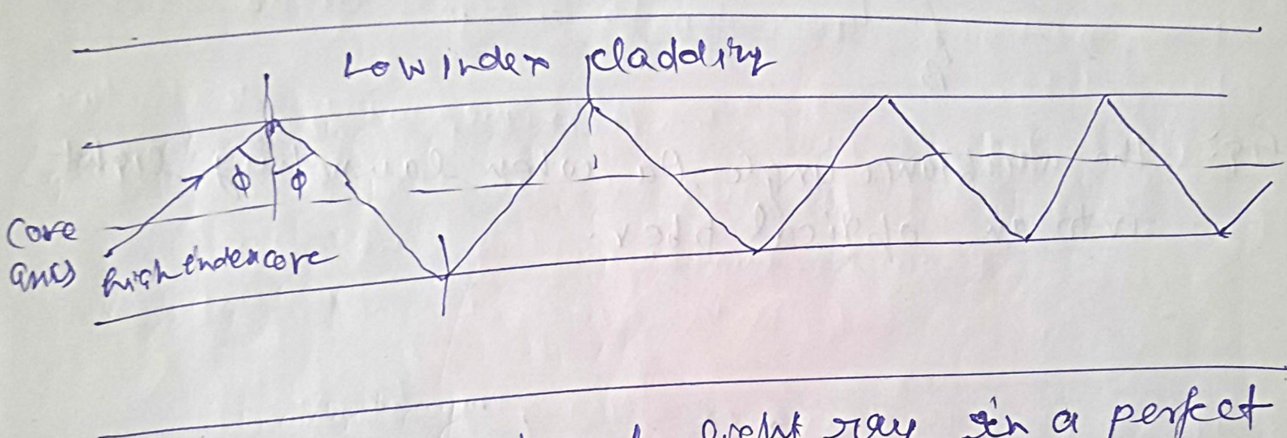


Fig: the transmission of light ray in a perfect optical fiber.

→ A series of total internal reflections at the interface of the silica core and slightly lower refractive index silica cladding.

$\phi > \phi_c$

⇒ the ray has an angle of incidence ϕ at the interface which is greater than the critical angle and is reflected at the same angle to the normal

→ the light ray is known as a meridional ray as it passes through the axis of the fiber core.

Acceptance Angle.

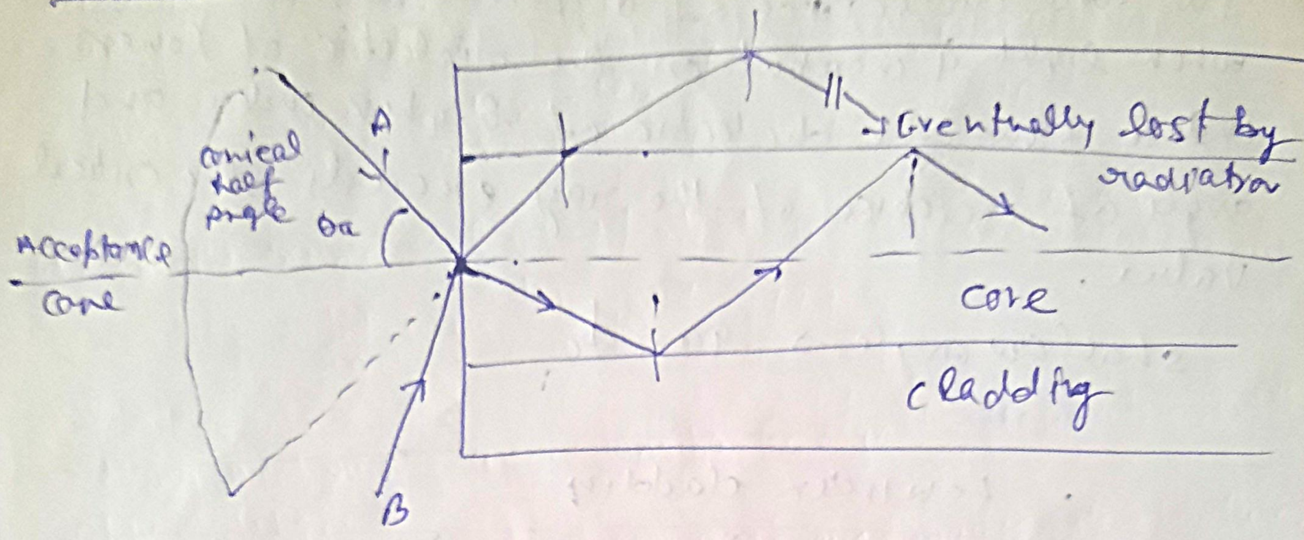


Fig: the acceptance angle θa when launching light into an optical fiber.

Acc-Angle | NA | Skew Rays | Cutoff wavelength | MFD | Spot size

(2)

Acceptance Angle -

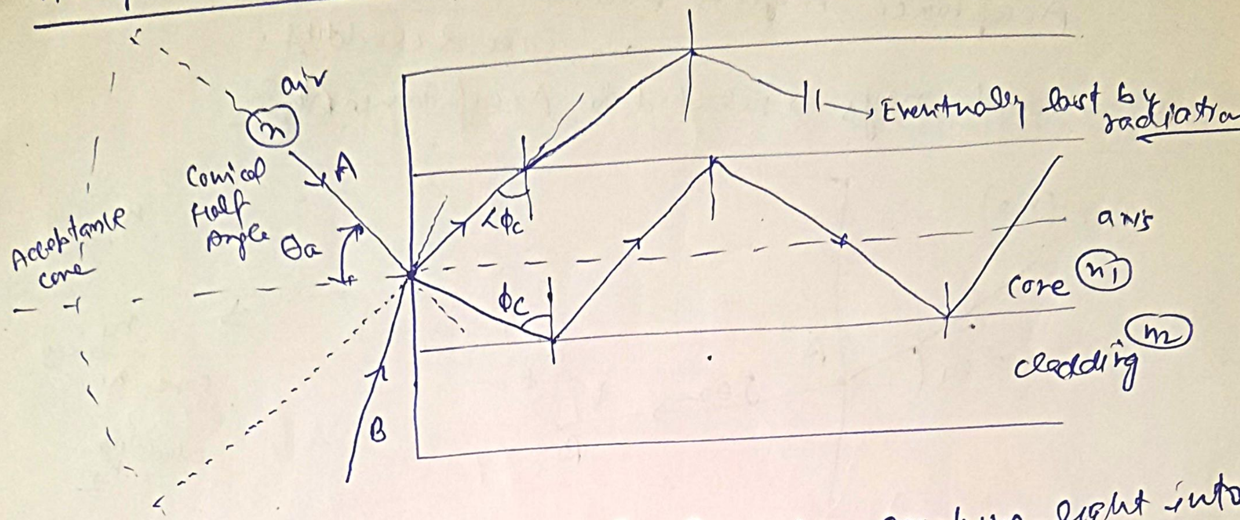


Fig: Acceptance angle θ_a when launching light into an optical fiber.

when ray A. incident Angle $> \theta_a \Rightarrow \phi < \phi_c \rightarrow$ not totally internally reflected.

↳ the incident ray B at an Angle greater than θ_a is refracted into the cladding and eventually lost by radiation.

For the total internal reflection of the ray, there must be a limit of $\theta \Rightarrow \theta \leq \theta_a$

θ_a is the max angle at which light may enter the fiber in order to propagated is referred to as acceptance Angle for the fiber.

Numerical Aperture -

Acceptance Angle → three Medium Involved air, core & cladding.

Meridional rays → related to Acceptance Angle

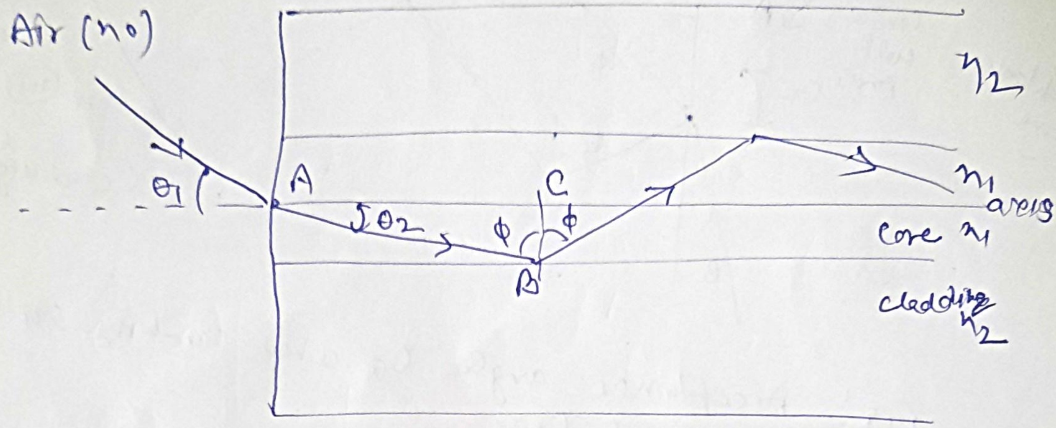


Fig: The ray path for a meridional ray launched into an optical fiber in air at an input angle less than Acceptance angle for the fiber.

Snell's Law → $n_0 \sin\theta_1 = n_1 \sin\theta_2$

$\phi > \phi_c$ at core cladding interface

$\phi = \frac{\pi}{2} - \theta_2 \Rightarrow \theta_2 = \left(\frac{\pi}{2} - \phi\right)$

$n_0 \sin\theta_1 = n_1 \sin\left(\frac{\pi}{2} - \phi\right)$

$= n_1 \cos\phi \Rightarrow n_0 \sin\theta_1 = n_1 \left[(1-\sin^2\phi)^{\frac{1}{2}}\right]$

$\sin^2\phi + \cos^2\phi = 1$

$\cos\phi = (1-\sin^2\phi)^{\frac{1}{2}}$

when $\phi = \phi_c$

then $\theta_1 = \theta_a$

$n_0 \sin\theta_a = n_1 (1-\sin^2\phi_c)^{\frac{1}{2}}$

$n_0 \sin\theta_a = (n_1^2 - n_2^2)^{\frac{1}{2}}$

$\sin\phi_c = \frac{n_2}{n_1} \Rightarrow \phi_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)$

NA (Numerical Aperture) is the important fiber parameter

$NA = n_0 \sin\theta_a = (n_1^2 - n_2^2)^{1/2}$

For air → $n_0 = 1$ ⇒ $\boxed{NA = \sqrt{n_1^2 - n_2^2}}$

Sometimes NA is given in terms of relative Refractive Index Difference Δ b/w the core & cladding

$\xrightarrow{\text{Index difference}}$ $\Delta n = n_1 - n_2$ | $\frac{\Delta n}{n_1} \rightarrow$ fractional index difference

$$\Delta = \frac{n_1^2 - n_2^2}{2n_1^2}$$

$$\boxed{\Delta \approx \frac{n_1 - n_2}{n_1} \text{ for } \Delta \ll 1}$$ → For step Index Fiber

NA of a step Index Fiber

$$\Rightarrow \boxed{NA = n_1 (2\Delta)^{\frac{1}{2}}}$$

Q. Consider a multimode silica fiber that has a core refractive index $n_1 = 1.480$ and a cladding index $n_2 = 1.460$ find (a) the critical Angle (b) numerical Aperture (c) the acceptance Angle.

Sol (a) $\phi_c = \sin^{-1}\left(\frac{n_2}{n_1}\right) = 80.5^\circ$

(b) $NA = \sqrt{n_1^2 - n_2^2} = 0.242$

(c) $n_0 = 1 \Rightarrow \theta_a = \sin^{-1} NA = \sin(0.242) = 14^\circ$

Q. Consider a multimode fiber that has a core refractive index 1.480 & core cladding index difference 2.0 percent ($\Delta = 0.020$) find (a) NA (b) the acceptance Angle (c) critical Angle

Sol ⇒ Step Index → n_1 → radius a = 1.480 (n_1)

$n_2 = n_1(1-\Delta) = \frac{n_2}{n_1} = 1-\Delta \Rightarrow \Delta = 1 - \frac{n_2}{n_1} = \frac{n_1 - n_2}{n_1}$

$\Delta \rightarrow$ core cladding index diff or index difference

$n_2 = n_1(1-\Delta)$ $\quad \Delta \simeq 0.01$

$\Delta = 1-3\%$ for MMF

$= 0.2-1.0\%$ for SMF

(i) $NA = n_1(\sqrt{2\Delta}) = 1.48\sqrt{2 \times 0.02}$

$= 0.296$

(ii) Acceptance Angle in Air $= \sin^{-1}(NA)$

$= 17.2°$

(iii) Critical Angle at core cladding Index

$\phi_c = \sin^{-1}\left(\frac{n_2}{n_1}\right) = \sin^{-1}(0.980)$

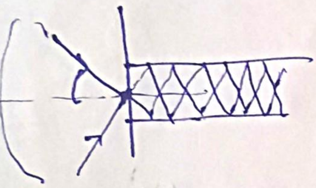
$= 78.50$

$n_2 = n_1(1-\Delta)$

$n_2 = 1.45$ | $n_1 = 1.48$

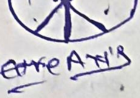
Meridional Rays & Skew Rays.

Meridional



1) Follow Total Internal Reflection
2) Passes through the core
3) All rays entering in the core through Axis within Acceptance Cone
4) TIR in Same Plane

5) Cross section of Core



Skew Rays →

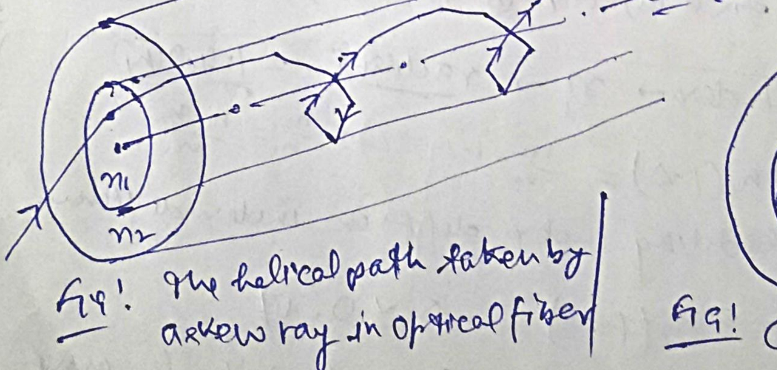


Fig: The helical path taken by a skew ray in optical fiber

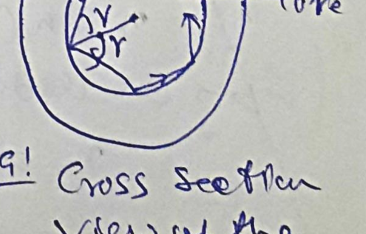


Fig: Cross section view of the fiber

The rays which are transmitted without passing through the fiber axis; follow a helical path through the fiber, are called skew rays.

→ γ is the angle b/w the projection of ray in two dimensions and the radius of the fiber core at the point of reflection.

→ The direction of the skew rays changes at each reflection at an angle 2γ.

→ Light input to the fiber : non-uniform
then skew rays will have smoothing effect on the transmission of light, giving more uniform output.

→ The amount of smoothing depends on the number of reflections encountered by the skew rays.

$\cos\phi = \frac{BT}{AB}$

γ - small $\cos\gamma \approx 1$

$\cos\gamma =$

$\sin\theta = \frac{AC}{AB}$

Acceptance Angle for skew rays →

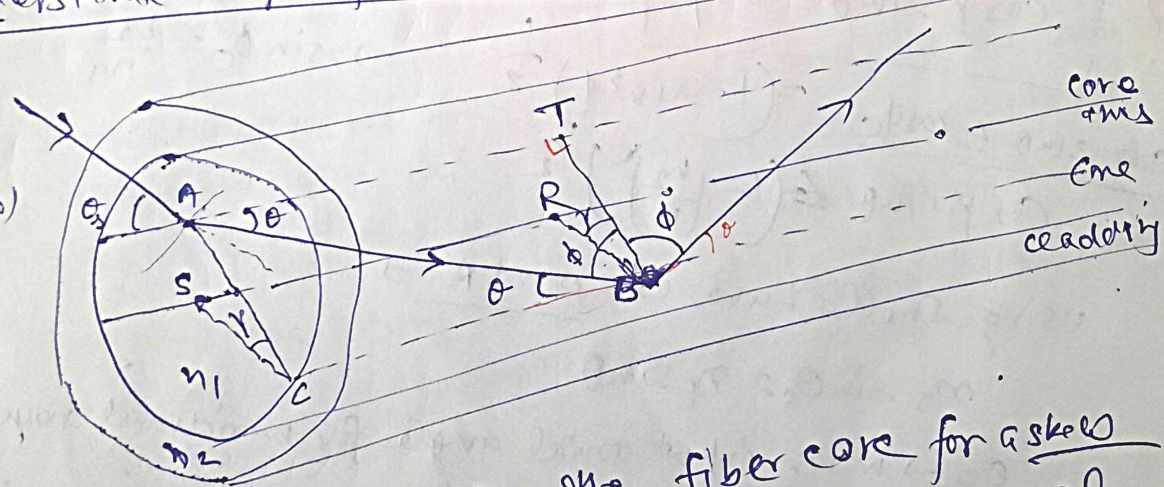


Fig: The ray path within the fiber core for a skew ray incident at an angle θ_s, to the normal at air-core interface.

The direction of the rays is in two perpendicular plane

- $\theta_s \rightarrow$ Angle of incident ray from the normal.
- A → incident point
- at air-core interface → incident ray A & refracted ray to B are in same plane
- At point B → $\phi_i = \theta_r \Rightarrow \phi \rightarrow$ since $\angle$ reflected & $\phi > \phi_c$ at core-cladding interface
- At point B → incident ray & reflected ray are in same plane.

$\gamma \rightarrow$ angle b/w the core radius and the projection of the ray on to a plane BRS normal to the core axis.

AB & BR radius ⇒ two perpendicular plane requiring multiplication by $\cos\gamma$ and $\sin\theta$.

Ray path

? $\boxed{\cos\gamma \sin\theta = \cos\phi}$ → $\sin\theta = \frac{\cos\phi}{\cos\gamma}$

at $\phi = \phi_c$

$\Rightarrow \sin\phi_c = \frac{n_2}{n_1}$

$= (1 - \sin^2\phi)^{\frac{1}{2}}$

$\cos\gamma \sin\theta \le \cos\phi_c$

$\cos\gamma \sin\theta \le \left(1 - \left(\frac{n_2}{n}\right)^2\right)^{1/2}$

using Snell's Law at point A →

$n_0 \sin\theta_a = n_1 \sin\theta$

$\theta_a \rightarrow$ max. input axial angle for meridional rays.

$\theta \rightarrow$ internal axial Angle

$\sin\theta_{as} = \frac{n_1}{n_0} \cdot \sin\theta = \frac{n_1}{n_0} \cdot \frac{\cos\phi_c}{\cos\gamma} = \frac{n_1}{n_0 \cdot \cos\gamma} \cdot \left(1 - \frac{n_2^2}{n_1^2}\right)^{1/2}$

$\theta_{as} \rightarrow$ maximum input Angle / Acceptance Angle for skew rays

The acceptance conditions for skew rays are—

$$n_0 \sin\theta_{as} \cos\gamma = \left(\frac{n_1^2 - n_2^2}{n_1^2}\right)^{1/2}$$

$$= (n_1^2 - n_2^2)^{1/2} = NA$$

$n_0 = 1$ for Air

$$\boxed{\sin\theta_{as} \cdot \cos\gamma = NA}$$

⇒ skew rays are Accepted at larger axial angle in a given fiber than meridional rays. depending upon the value of $\cos\gamma$.

For meridional rays → $\cos\gamma$ = unity = 1
& $\theta_{as} = \theta_a$

θ_a → maximum conical half angle for the acceptance of meridional rays.
It defines the minimum input angle for skew rays.

⇒ Skew rays tend to propagate only in the annular region near the outer surface of core. and do not fully utilized the core as a transmission medium.

⇒ These rays are complementary to meridional rays and increase the light-gathering capacity of the fiber.

Q. An optical fiber in air has an NA of 0.4. Compare the acceptance angle for meridional rays with that for skew rays which change direction by 100° at each reflection.

$n_0 = 1 \Rightarrow \theta_a = \sin^{-1}(NA) = \sin^{-1}(0.4)$

$\theta_a = 23.6^\circ$

The skew rays change direction by 100° at each reflection

so $\gamma = \frac{100}{2} = 50^\circ$

Hence acceptance angle for skew ray is

$\theta_{as} = \sin^{-1}\left(\frac{NA}{\cos\gamma}\right) = \sin^{-1}\left(\frac{0.4}{\cos 50^\circ}\right) = 38.5^\circ$

$\theta_{as} > \theta_a \Rightarrow 38.5 - 23.6$

$= 14.9 \simeq 15^\circ$

$\theta_{as} = \theta_a + 15^\circ$

⇒ when the light input to the fiber is at an angle to fiber axis, γ will vary from (zero to 90°) — for meridional ray at the core cladding interface.

At gives acceptance of skew rays over a conical half angle of $\pi/2$ radians.

skew rays in graded index fiber

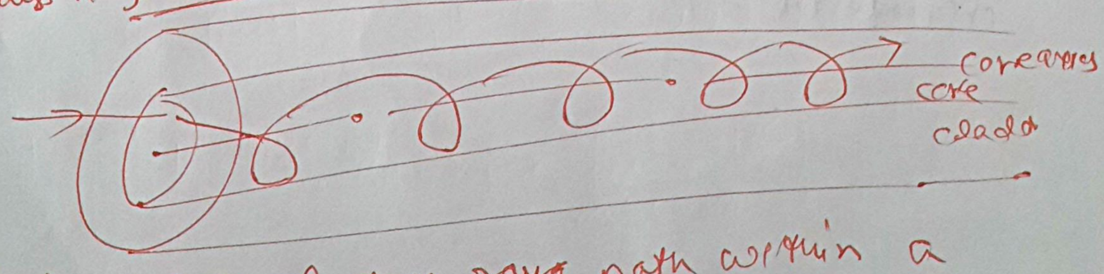


Fig: A helical skew rays path within a graded index fiber

Cutoff Wavelength -

Relation b/w normalized freq. & NA

$$V = \frac{2\pi}{\lambda} \cdot a (NA) = \frac{2\pi}{\lambda} \cdot a\, n_1 (2\Delta)^{1/2} \quad \text{(1)}$$

$\Delta \to$ relative refractive index difference

$$\lambda = \frac{2\pi\, a n_1 (2\Delta)^{1/2}}{V}$$

for cutoff $\to$ $\underset{\text{freq}}{V_c} \to \underset{\text{wav}}{\lambda_c}$

$$\lambda_c = \frac{2\pi}{V_c} \cdot a n_1 (2\Delta)^{1/2} \quad \text{(2)}$$

$\lambda_c \to$ wavelength above which a particular fiber becomes single moded.

from 1 & 2

$$\frac{\lambda_c}{\lambda} = \frac{V}{V_c}$$

for step index fiber $V_c = 2.408$

so cutoff wavelength

~~$\lambda_c = 2.405$~~

$$\lambda_c = \frac{V\lambda}{2.405}$$

Q Determine the cutoff wavelength for a step index fiber to exhibit single-mode operation when the core refractive index and radius are 1.46 & 4.5 um respectively, with the relative refractive index being 0.25%.

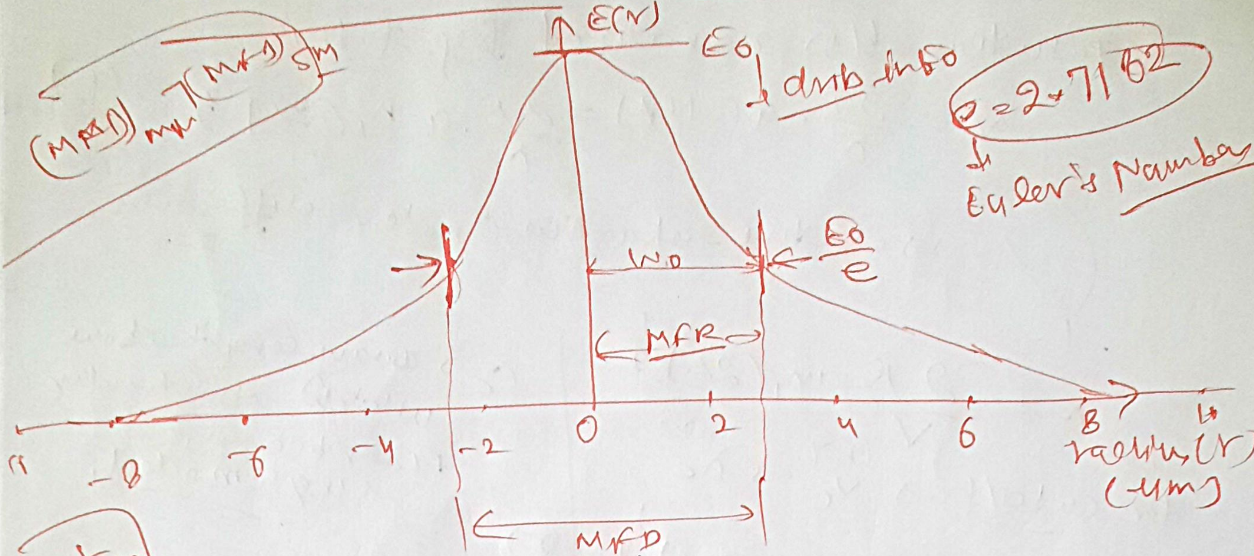
$$V\lambda = \frac{2\pi}{\cancel{V}} \cdot a n_1 (2\Delta)^{1/2} \times V$$

$$\lambda_c = \frac{V\lambda}{2.405} = \frac{2\pi \times 1.46 \times 4.5 \times (2 \times .0025)^{1/2}}{2.405}$$

$= 1214$ nm

Hence the fiber is single moded to a wavelength of 1214 nm

Mode Field Diameter & spot size



2 × 1 MFD

= 2W0

Fig: Field Amplitude Distribution E(r) of the fundamental mode in a singlemode fiber illustrating MFD & spot size.

Spot size

Mode field Radius → is equal to the distance from the center at which field strengths are reduces to $\frac{1}{e}$ of their maximum value.

Mode field Diameter → It is defined as twice of the mode field radius

→ is ~~taken~~ defined as the distance b/w the two opposite points on ~~the field strength~~ $\left(\frac{1}{e} = 0.37\right)$ of the field signal Amplitude

Spot size

corresponding power points = $\left(\frac{1}{e^2}\right) = 0.135$ of the Power

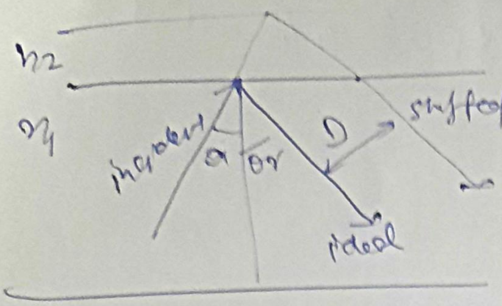
intensity Drops by → e^2

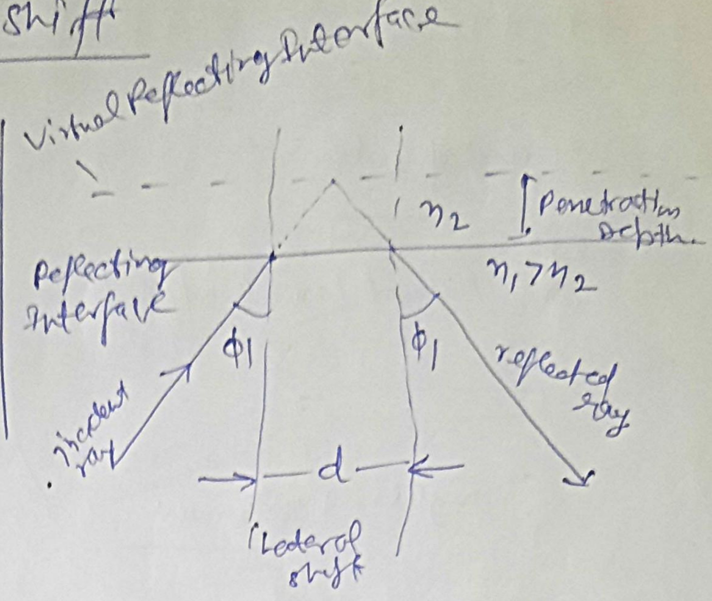
= −8.65 dB at the distance from the centre

spot size = $\frac{\text{Intensity}}{e^2} = \frac{E_0}{e^2}$

Petermann's II def

Goos - Haenchen shift





→ Reflected beam is shifted laterally from trajectory predicted by single ray theory Analysis. This lateral displacement due to shifting is called Goes Haenechen shift:

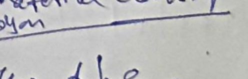
Fig: Lateral displacement of a light Beam on reflection at a dielectric interface.

Mode Coupling / Mode Mixing / Mode Conversion | mode of propagation light changed

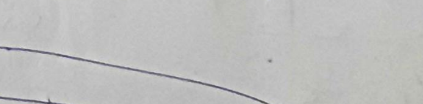
MMP

Reason → 1) The deviation of the fiber axis from straightness →

2) Variation in the core diameter



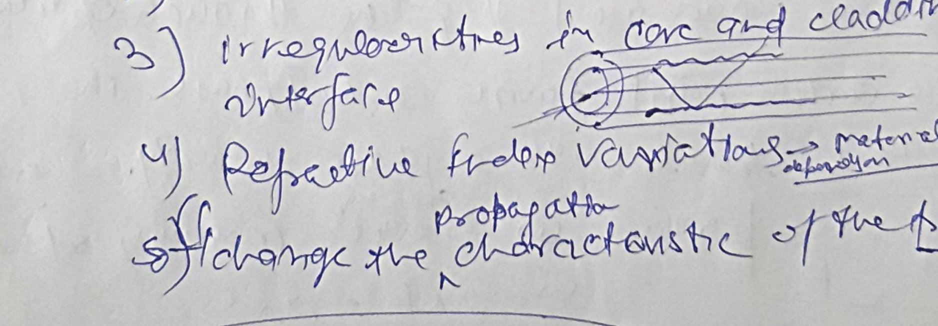
3) irregularities in core and cladding interface



4) Refractive Index Variations → material density dispersion

5) change the propagation characteristic of the fiber

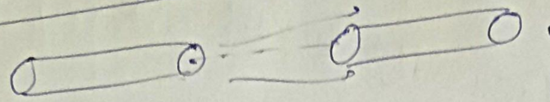
(I)



(b) fiber loop

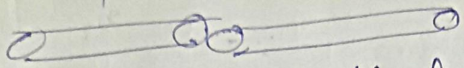
Types of Fiber Misalignments →

(i)



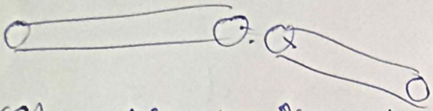
(a) end separation

(ii)



(b) Lateral / axial offset

(iii)



(c) angular Misalignment

Loss of Information

⇒ The ray does not maintain the same angle with axis

⇒ change in propagation Mode of the light

→ Individual modes do not normally propagate throughout the length of the fiber without large energy transfer to the adjacent modes. This mode conversion is known as mode coupling / mixing.

Phase Velocity & Group Velocity in optical fiber.

↓

when optical waves are propagating through optical fiber, there are certain point having constant phase. These points of constant phase travels with a phase velocity.

$$\boxed{v_p = \frac{\omega}{\beta}}$$

where ω → ~~radial~~ angular frequency of wave / angular velocity

$\omega = 2\pi f = 2\pi \frac{c}{\lambda}$

β → propagation constant

$$v_p = \frac{c}{n_1}$$

⇒ However it is impossible in practice to produce perfectly mono-chromatic light wave.

→ single wavelength light

Group velocity - Optical waves are travelling as wave packets. These wave-packets have group ~~velo~~ velocity v_g.

$$v_g = \frac{d\omega}{d\beta} = \frac{c}{N_g}$$

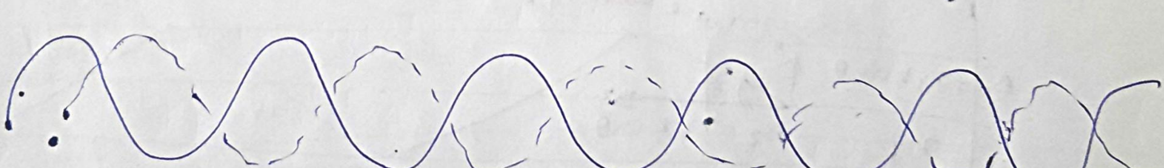
where c = velocity of light = 3×10^{8} m/s

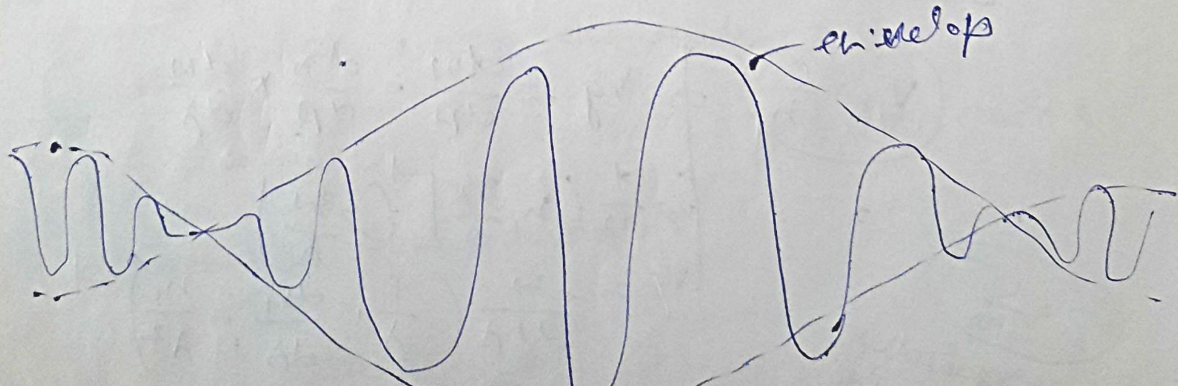
N_g → Group index of the guide

→ Light energy is generally composed of a sum of plane wave components of different frequencies

so a group of waves propagate and forms a packet of waves.

The wave packets does not travel at a phase velocity of individual waves but move ~~with~~ a group velocity.





⇒ The formation of a wave packet from the combination of two waves nearly equal freq. The envelope of the wave package or group of waves travel at a group velocity.

β (Propagation Constant) → The measure of the change in amplitude and phase per unit distance is called the propagation constant.

If propagation occur in infinite medium of n_1

then $\boxed{\beta = n_1 \frac{2\pi}{\lambda} = n_1 \frac{\omega}{c}}$ $\quad \omega = \frac{c\beta}{n_1}$

c → velocity of light in free space

$v = n\lambda$

$\lambda = \frac{v}{n} = \frac{v}{f}$

$\beta = n_1 \times \frac{2\pi f}{c} = \frac{n_1 \omega}{c}$

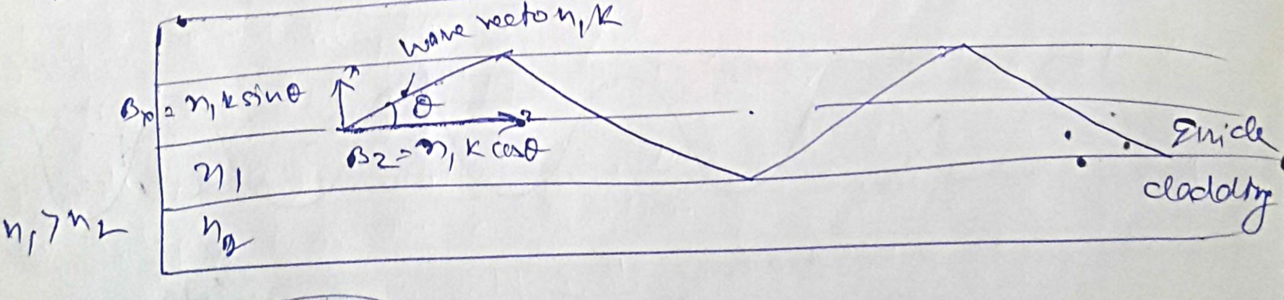
$k = \frac{2\pi}{\lambda}$

λ → optical wave length in vacuum

→ magnitude of propagation vector / vacuum phase propagation constant / free space wave number

$\beta_z = n_1 k \cos\theta = n_1 \frac{2\pi}{\lambda} = \frac{n_1 \omega}{c}$

The component of ~~wave~~ phase propagation constant in z-direction. β



$V_p = \frac{c}{n_1}$

$\Rightarrow V_g = \frac{d\omega}{d\beta} = \frac{d\lambda}{d\beta} \times \frac{d\omega}{d\lambda}$

$V_g = \frac{d}{d\lambda}\left(n_1 \frac{2\pi}{\lambda}\right)^{-1} \left(-\frac{\omega}{\lambda}\right)$

$V_g = \frac{-\omega}{2\pi\lambda}\left(\frac{1}{\lambda} \cdot \frac{dn_1}{d\lambda} - \frac{n_1}{\lambda^2}\right)^{-1}$

$V_g = \frac{c}{\left(n_1 - \lambda \frac{dn_1}{d\lambda}\right)} = \frac{c}{N_g}$

$V_g = \frac{c}{N_g}$

N_g group index of guide

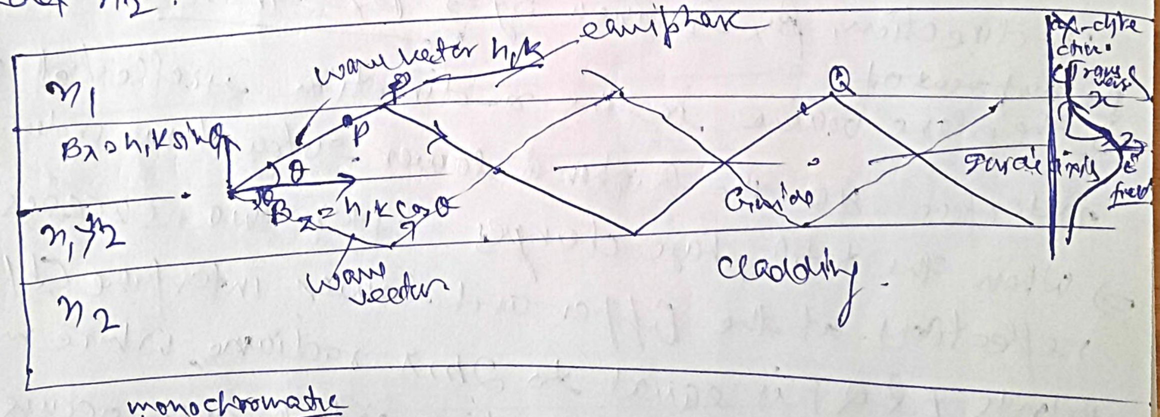
* Modes in a planer guide .

* Electromagnetic mode theory for optical propagation
* Phase shift with total Internal reflection
* Evanescent field.
* Cylindrical Fiber Modes

Modes in a planar waveguide

1) The planar guide is the simplest form of optical waveguide
2). It consists of a slab of dielectric with refractive index n_1, sandwiched b/w two regions of lower refractive index n_2.

3)



Consider a monochromatic wave propagating in the direction of the ray path within the guide.

$\Rightarrow \quad \gamma = \alpha + j\beta$

attenuation constant

phase constant

prop. constant

β - prop. constant

→ As the refractive index within the guide is n_1, the optical wavelength in this region is reduced to λ/n_1, while the vaccum propagation constant is increased to $n_1 k$.

⇒ When θ is the angle b/w wave propagation vector or equivalent ray and guide axis. The plane wave can be resolved into two component plane waves propagating in the z & x direction.

⇒ The component of the phase propagation constant in the z-direction β_z is given by: $\beta_z = n_1 k \cos\theta$

⇒ The component of the phase propagation constant in the x-direction, β_x is given by → $\beta_x = n_1 k \sin\theta$.

⇒ The component of the plane wave in the x-direction is reflected at the interface b/w the higher & lower refractive index media.

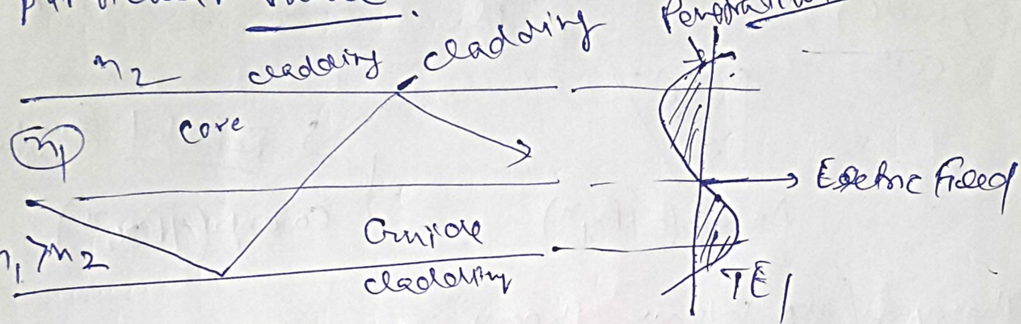
⇒ When the total phase changes after two successive reflections at the upper and lower interface (b/w the points P & Q) is equal to $2m\pi$ radians, where m is an integer then the constructive interference occurs and a standing wave is obtained in the x-direction

⇒ Here interference forms the lowest order (m=0) standing wave, where the electric field is maximum at the centre of guide decaying towards zero at the boundary b/w the guide & cladding.

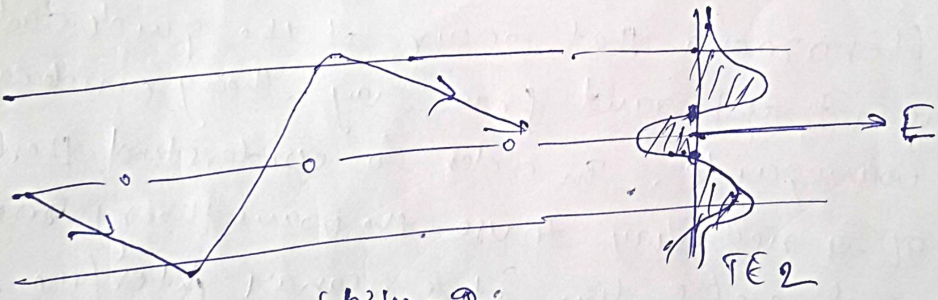
⇒ Thus the optical wave is effectively confined within the guide and the electric field distribution in the x-direction does not change as the wave propagates in the z-direction

⇒ The stable field distribution in the x direction with only a periodic z dependence is known as mode

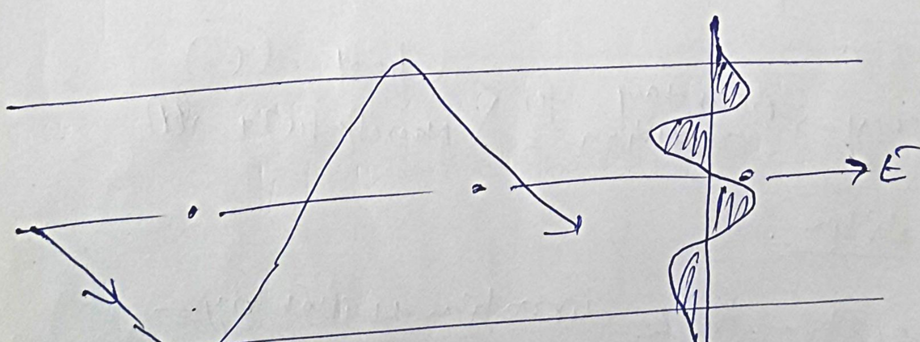
⇒ A specific mode is obtained only when the angle b/w the propagation vectors or rays and the interface have a particular value.



(a) m = 1

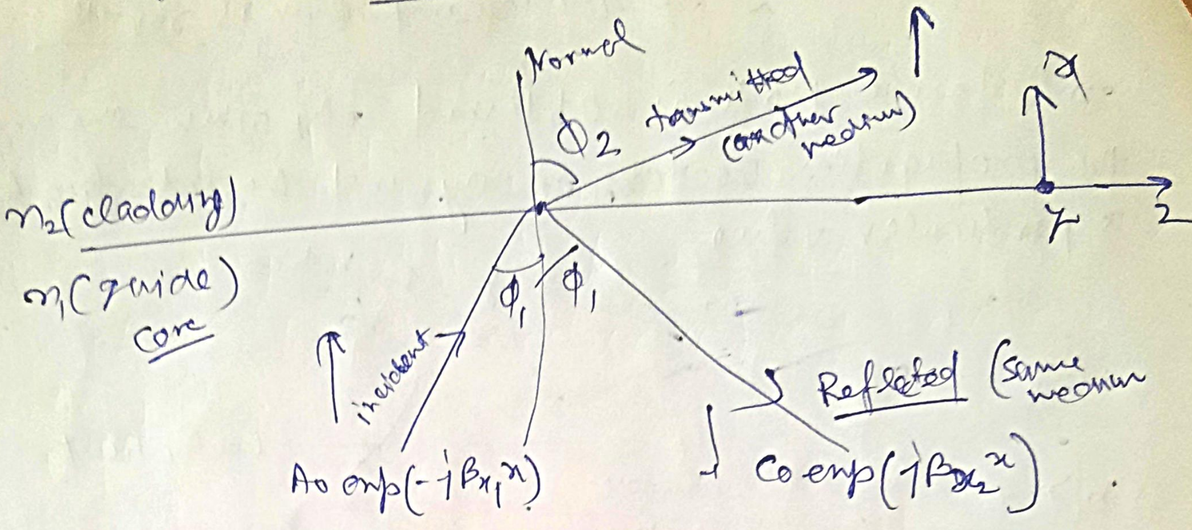


(b) m = 2



(c) m = 3

Phase shift with total Internal Reflection (TIR)



ray theory & / wave theory.

⇒ Certain phenomena that occur at the guide-cladding interface are not apparent from ray theory consideration of optical waveguide. In order to understand these phenomena. It is necessary to use the wave theory model for total internal reflection at a planar interface,

⇒ n_2 cladding

n_1 core

ray theory → Boundary condition for E field · (E) / Magnetic field → (H)

Boundary Condition.

- The wave propagation in Z-direction is given by -

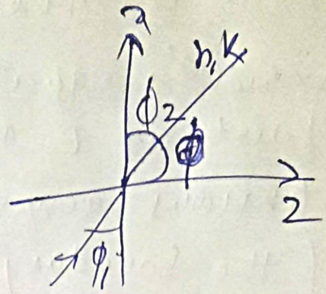
$\rightarrow \exp j(\omega t - \beta z)$ | $\beta = \frac{2\pi}{\lambda}$ (Lossless Medium)

$\gamma = \alpha + j\beta$ → (Lossy Medium)

propagation constant in x-direction for the guide $\beta_{x_1} = n_1 k \cos\phi_1$

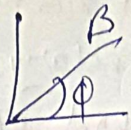
Propagation constant in x-direction for cladding

$$\beta_{x_2} = n_2 k \cos\phi_2$$



for free space

$$\left\{ \begin{aligned} \beta_z &= k\cos\phi \\ &= \beta\cos\phi \end{aligned} \right.$$



$\beta \lesssim k$
$= n_1 k$

For medium $\beta = n_1 k \cos\phi$

Thus the three waves in the waveguide are incident, transmitted and reflected with amplitude A, B, C respectively, will have the form →

$$A = A_0 \exp-(i\beta_{x_1} x)\exp(i(\omega t - \beta z))$$

$$B = B_0 \exp-(i\beta_{x_2} x)\exp(i(\omega t - \beta z)).$$

$$C = C_0 \exp-(i\beta_{x_1} x)\exp(i(\omega t - \beta z))$$

By using simple trigonometric relationship —

$$\cos^2\phi + \sin^2\phi = 1$$

$$\beta_{x_1}^2 = (n_1 k \cdot \cos\phi_1)^2 = n_1^2 k^2 \cos^2\phi_1 = n_1^2 k^2 (1 - \sin^2\phi_1)$$

$$= n_1 k^2 - \underbrace{n_1^2 k^2 \sin^2\phi_1}_{\beta}$$

$$\beta_{x_1}^2 = (n_1^2 k^2 - \beta^2) = -\xi_1^2$$

1 → core
2 → cladding

Similarly

$$\beta_{x_2}^2 = (n_2 k^2 - \beta^2) = -\xi_2^2$$

⇒ When an electromagnetic wave is incident upon an interface between two dielectric media.

- Maxwell's equation require tho both the tangential components of E and H and normal component of D are continuous across the boundary. If

If the boundary is defined at $x=0$ we may consider the case of the transverse electric (TE) and transverse magnetic (TM) modes.

→ Normal B.C → $A_0 + C_0 = B_0$ — ①

→ Tangential B.C →

$-\beta_{x_1} A_0 + \beta_{x_2} C_0 = -\beta_{x_2} B_0$ — ②

→ incident + reflected = Transmitted

On solving ① & ② →

$$C_0 = A_0\left(\frac{\beta_{x_1}-\beta_{x_2}}{\beta_{x_1}+\beta_{x_2}}\right)$$

$$= A_0 \Gamma_{ER}$$

Γ → tow

$$B_0 = A_0\left(\frac{2\beta_{x_1}}{\beta_{x_1}+\beta_{x_2}}\right)$$

$$= A_0 \Gamma_{ET}$$

continuity equation for E-field

$D_{n_1} = D_{n_2}$

$D = \varepsilon E$

$D \propto E$

$E_{n_1} = E_{n_2}$

normal

normal component of E field in n_1 medium = normal compo. of E in n_2 me

$H_{t_1} = H_{t_2}$

tangential component

$$\frac{C_0}{A_0} \quad \tau_{ER} = \frac{\beta_{x_1}-\beta_{x_2}}{\beta_{x_1}+\beta_{x_2}}$$

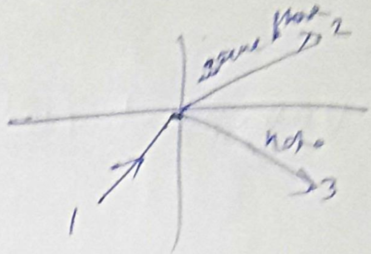
= reflection coefficient

τ_{ET} → Transmitted coefficient

$$\frac{B_0}{A_0} = \left(\frac{2\beta_{x_1}}{\beta_{x_1}+\beta_{x_2}}\right)$$

When both β_{x_1} & β_{x_2} are real. It is clear that the reflected wave C is in the phase with the incident wave A. But after critical angle for total internal reflection, β_{x_2} becomes imaginary but β_{x_1} remains real.

So. $C_0 = A_0 \left(\dfrac{\beta_{x_1} + j\xi_{2}}{\beta_{x_1} - j\xi_{2}} \right)$



$= A_0 \exp(2j\delta_E$

$\boxed{\delta_E \Rightarrow \text{phase shift}}$

Now, here, we observe, there is a phase shift on the reflected wave relative to the incident wave. This is signified by δ_E. which is given by —

$$\boxed{\tan\delta_E = \frac{\xi_2^2}{\beta_{x_1}}}$$ phase shift

⇒ A similar analysis may be applied for TM mode at the interface. which leads to —

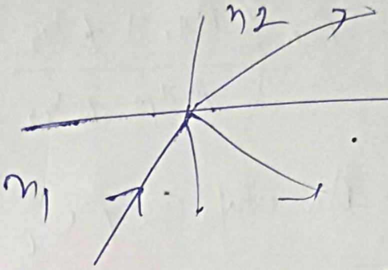
$$C_0 = A_0 \left(\frac{\beta_{x_1} n_2^2 - \beta_{x_2} n_1^2}{\beta_{x_1} n_2^2 + \beta_{x_2} n_1^2} \right) = A_0 \tau_{TR}$$

$$B_0 = A_0 \left(\frac{2\beta_{x_1} n_2^2}{\beta_{x_1} n_2^2 + \beta_{x_2} n_1^2} \right) = A_0 \tau_{MT}$$

$$C_0 = A_0 \exp(2j\delta_m) \Rightarrow \tan\delta_M = \left(\frac{n_1}{n_2}\right)^2 \tan\delta_E$$

Thus the phase shift obtained on total internal reflection is dependent upon both the angle of incidence and the polarization (either TE or TM) of the radiation.

Evanescent field



Before the critical angle for the TIR is reached, there is only partial reflection. The field in the cladding is given by →

$$B = B_0 \exp -(j\beta_{x_2} x) \exp j(\omega t - \beta z)$$

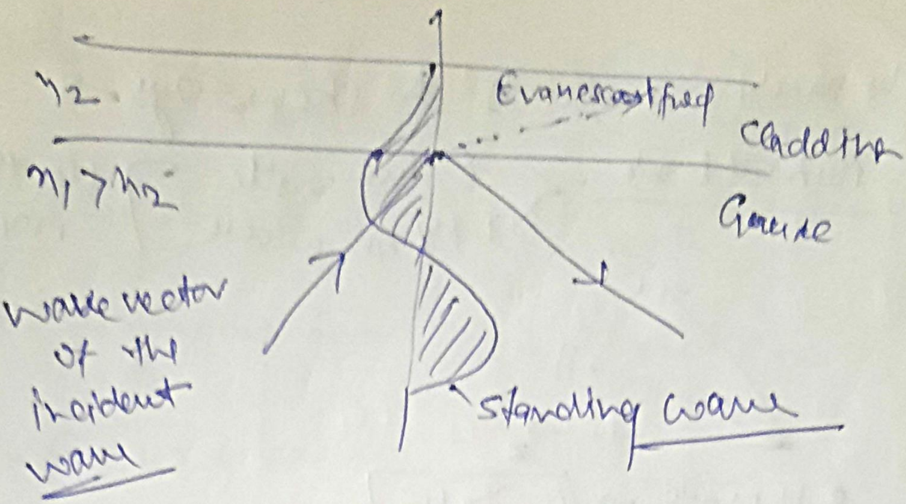
However when the TIR is reached, β_{x_2} becomes imaginary, transmitted wave in cladding becomes -

$$B = B_0 \exp(-\xi_2 x) \exp j(\omega t - \beta z)$$

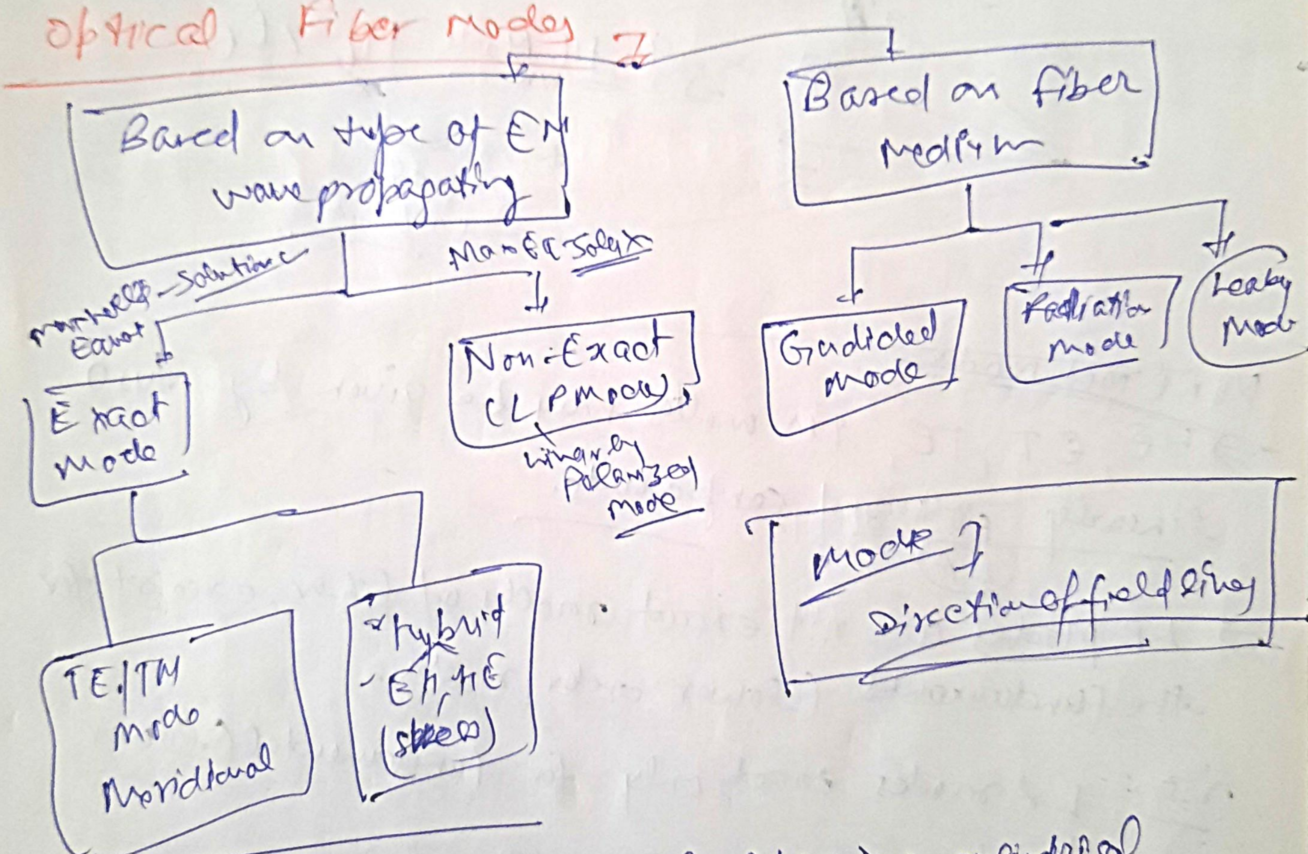
Thus the amplitude of the field in the cladding is observed to decay exponentially in x-direction. Such a field, having an exponentially decaying amplitude, is referred to as an evanescent field.

→ A field of this type stores energy and transport it in the direction of propagation (z) but does not transport energy in the transverse direction (x).

Nevertheless, the existence of an evanescent field beyond the plane of reflection in the lower index medium indicates that optical energy is transmitted into cladding.

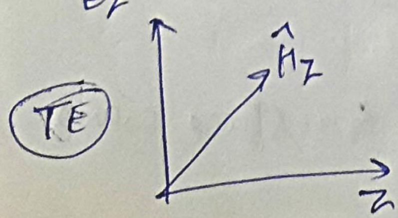


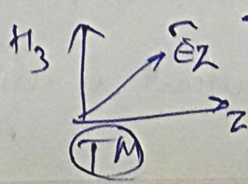
Optical Fiber Modes



Exact Modes → 1) The optical fiber is a cylindrical waveguide
2) The modes corresponds to Meridional rays: TE_{nm}, TM_{nm} mode.

$E_z = 0$

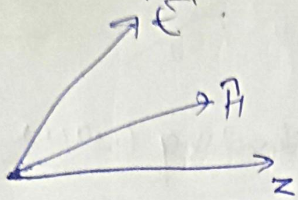




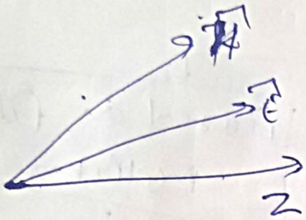
~~Non Exact~~ → Mode

The modes corresponds to skew rays →

Hybrid Mode - < EH_{nm} mode / HE_{nm} mode] Neither E & nor H = 0



In EH_{nm} Mode → $\boxed{E_t > H_t}$



⇒ In HE_{nm} Mode $H_t > E_t$

transverse magnetic component.

Non Exact mode

⇒ HE, ET, TE, TM modes may be given by two linearly polarized components. (LP)

⇒ LP Modes are not exact modes of fiber, except for the fundamental (lower order) modes.

i.e. LP modes exact, only for fundamental (lower order) modes.

⇒ If Δ = small ($\Delta \ll 1$) then EH & HE modes have identical propagation constant such modes are said to be Degenerate Modes.

⇒ Propagation constant is also called as free space wave number

Relation b/w LP & Traditional HE, EH, TE & TM Mode

⇒ LP_{0m} is derived from HE_{1m} mode

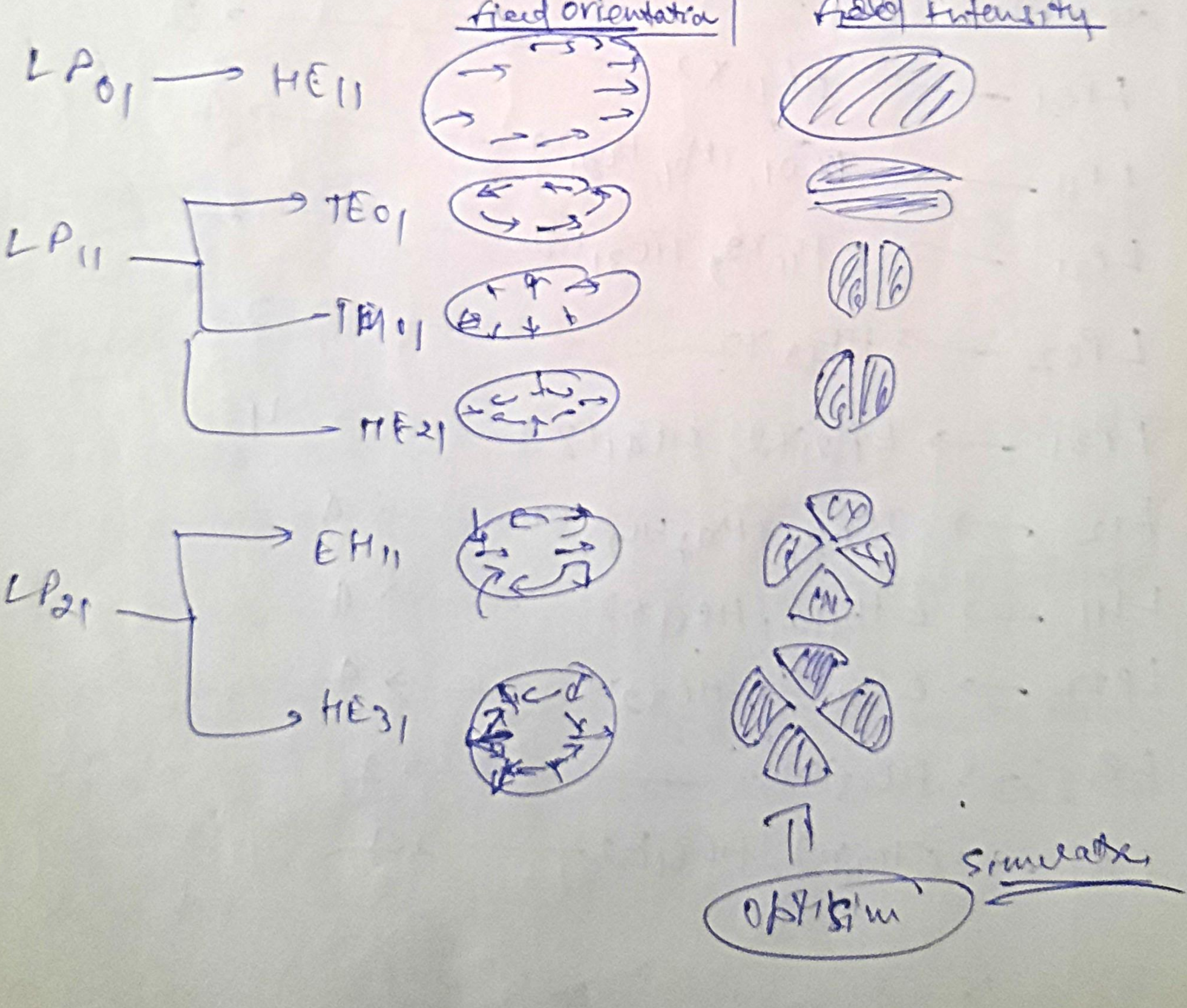
⇒ LP_{1m} is derived from TE_{0m}, TM_{0m} HE_{2m}

⇒ LP_{nm} {for $n \geq 2$} ⇒ $HE_{(n+1)m}$ & $EH_{(n-1)m}$

Composition of the Lower-order Linearly Polarized mode

LP mode designation	Traditional-mode designation & number of modes	no of degenerate modes
LP_{01} →	$HE_{11} \times 2$	→ 2
LP_{11} →	TE_{01}, TM_{01}, $HE_{21} \times 2$	→ 4
LP_{21} →	$EH_{11} \times 2$, $HE_{31} \times 2$	→ 4
LP_{02} →	$HE_{12} \times 2$	→ 2
LP_{31} →	$EH_{21} \times 2$, $HE_{41} \times 2$	→ 4
L_{12} →	TE_{02}, TM_{02} $HE_{22} \times 2$	→ 4
L_{41} →	$EH_{31} \times 2$, $HE_{51} \times 2$	→ 4
LP_{22} →	$EH_{12} \times 2$, $HE_{32} \times 2$	→ 4
LP_{03} →	$HE_{13} \times 2$	→ 2
LP_{51} →	$EH_{41} \times 2$, $HE_{61} \times 2$	→ 4

LP Mode	Exact mode
LP_{01}	HE_{11}
LP_{02}	HE_{12}
LP_{11}	$TE_{01}, TM_{01}, HE_{21}$
LP_{21}	HE_{31}, EH_{11}
LP_{31}	HE_{41}, EH_{21}
$LP_{nm} (n>1)$	$HE_{(n+1)m}, EH_{(n-1)m}$
LP_{1m}	$TE_{0m}, TM_{0m}, HE_{2m}$

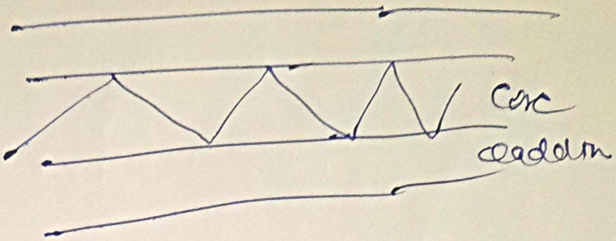


Based on the medium of propagation

1) Guided Mode

$\theta_i < \theta_a$

Loss: X

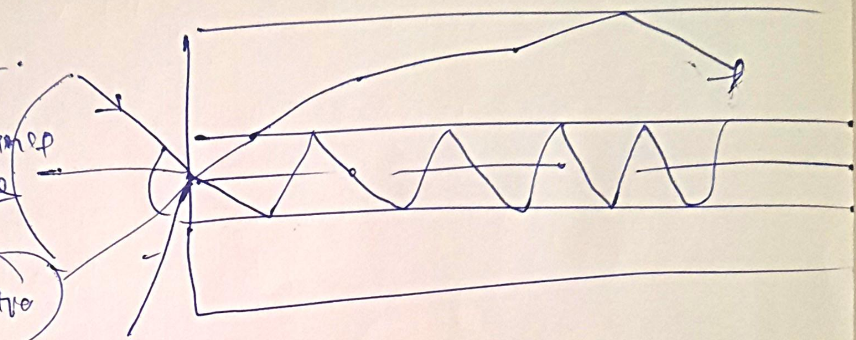


2) Radiation Mode

$\theta_i > \theta_a$

Acceptance cone

Loss of Information



3) Leaky Mode

$\theta_i > \theta_a$

Acceptance cone